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14. ABSTRACT
In this paper, we report on the initial development of the component capabilities, especially the imaging capabilities, in Computer Vision, Autonomy, and Teleoperation for the long-term goal of operating a team of robots for purposes of surveillance or materiel transport. The team is a collection of ?ve to ?fty monocular mobile robots that are jointly controlled by a single user with a joystick. Each robot communicates with nearby robots, could sense the terrain in its vicinity proprioceptively, and can coordinate with

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Report Title

Teleoperation of a Team of Robots with Vision

ABSTRACT

In this paper, we report on the initial development of the component capabilities, especially the imaging capabilities, in Computer Vision, Autonomy, and Teleoperation for the long-term goal of operating a team of robots for purposes of surveillance or materiel transport. The team is a collection of
?ve to ?fty monocular mobile robots that are jointly controlled by a single user with a joystick. Each robot communicates with nearby robots, could sense the terrain in its vicinity proprioceptively, and can coordinate with other robots to avoid obstacles as well as friendly assets.
In this e?ort, we focussed on the image sensing opportunities provided by such a team of monocular mobile robots and the computer vision capabilities required to exploit those opportunities. No e?ort was expended on SLAM, and little on control.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

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(2) Amit Bhatia and Wesley Snyder, Stacked Integral Image, IEEE International Conference on Robotics and Automation, Anchorage, May, 2010.

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(d) Manuscripts

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Patents Submitted

Patents Awarded

Graduate Students

NAME	PERCENT_SUPPORTED
Amit Bhatia	0.25
Stuart Heinrich	0.25
FTE Equivalent:	**0.50**
Total Number:	**2**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Wesley Snyder	0.15	No
Griff Bilbro	0.10	No
FTE Equivalent:	**0.25**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

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Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Amit Bhatia

Total Number: **1**

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

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Inventions (DD882)

Teleoperation of a Team of Robots with Vision

Final Report

Wesley Snyder and Griff Bilbro
co-principal investigators

November 1, 2010

A report to the Army Research Office on award W-911NF-09-1-0458.

Executive Summary

Objective The authors proposed to develop mathematics and algorithms which will allow a single operator to teleoperate a team of robots, using information obtained by cameras on the robots.

Approach

This STIR project had several components:

- Solve the Self Localization problem relative to a local coordinate system, using mutual distance information.
- Determine the Field of View of each camera, using the pose information already determined.
- Determine which cameras are on the leading edge.
- Construct the panoramic view and provide that view to the operator.
- From the joystick command, construct a force field and distribute that force to each robot.
- For each robot, make an appropriate movement.
- Provide feedback to the user in the event of unfulfillable motion commands.

Relevance This proposal is in response to the ARO Broad Agency Announcement, Sections 5.2 and 3.5.

Control of a team of robots by a single operator is particularly important in the urban warfare missions envisioned by the Future Combat Systems program. Such teams are expected to participate in a variety of surveillance and materiel transport missions.

The need for work in this area is exemplified by the fact that the U.S. Military has one component directly addressing problems similar to those addressed in this proposal, the “Multi-robot Operator Control Unit” in development at the Space and Naval Warfare Systems Center, San Diego[1]. Control of multiple robots in Army Applications is also being addressed by the SWARMS MURI at the University of Pennsylvania, as well as projects within the Army Tank Automotive Research and Development Center and the Army Research Laboratory.

Contents

1. Introduction

In this paper, we report on the initial development of the component capabilities, especially the imaging capabilities, in Computer Vision, Autonomy, and Teleoperation for the long-term goal of operating a team of robots for purposes of surveillance or materiel transport. The team is a collection of five to fifty monocular mobile robots that are jointly controlled by a single user with a joystick. Each robot communicates with nearby robots, could sense the terrain in its vicinity proprioceptively, and can coordinate with other robots to avoid obstacles as well as friendly assets.

In this effort, we focussed on the image sensing opportunities provided by such a team of monocular mobile robots and the computer vision capabilities required to exploit those opportunities. No effort was expended on SLAM, and little on control.

2. Objective

Most of the effort on this project involved the particular subproblem: **Automatic composition of a panoramic mosaic.** The operator must be able to naturally specify a direction to observe, the corresponding subset of individual cameras of robots on the periphery facing the specified direction must be identified, and the information in the selected images must be fused for presentation to the human operator in a way which the human can readily grasp and conveniently use.

To fully accomplish control of a robot team, we would have to incorporate advanced sensing with path planning and control. That implies a list of objectives too ambitious to be completed by a small team of researchers in a brief interval of time; this study therefore limited itself to the imaging component.

3. Approach

While recognizing the importance of fusing of sensing and control, we focussed on the sensor suite here rather than on any particular part of control, guidance, path planning, or path following. Other groups are working on these latter problems and we hope to take advantage of their results if we are successful in demonstrating the computer vision capabilities that we have proposed above, especially panorama composition, which we regard as a milestone accomplishment to justify subsequent effort. To our knowledge, no one is working on any such real-time, joystick-directed composition of panorama mosaics from an optimally collected subset of a collection of monocular mobile robots which are, in general, dispersed in an arbitrary, ad-hoc distribution of locations.

We developed an algorithm for determining how to position and direct cameras in such a way that the entire external field of view is guaranteed to be observed. The mathematical details are presented in the Results section. This involved determining camera pose (algorithm 1) and camera focal length (algorithm 2).

4. Background

Though the problem of making sense of information from distributed cameras is relatively new, already there has been one international conference on the topic (held in Vienna in September of 2007), and is about to be a second (Stanford in September, 2008). The problem requires some sort of consistency analysis, for example, using consistent labeling [2] or game theory [3]. As described in the appendix, our approach also uses consistency, an extension of the generalized Hough transform[4].

To construct the panoramic view envisioned, one might begin by evaluating the applicability of image stitching algorithms. Xing and Miao [5] use SIFT features in a way similar to the way our SKS [6] uses local neighborhoods. The reader is referred to [7] for an excellent review of the stitching literature, however, in this project, we determined an alternative approach to stitching, which will be discussed in the Results section of this report.

The concept of Visual SLAM is also relatively new: There was a workshop on Visual SLAM at the Intelligent Robotic Systems Conference in 2007 (San Diego, October, 2007). Although we mention the word, this project is, in itself, not a project in SLAM.

Because we have a human in the loop, we do not have to solve the SLAM problem globally. However, we must maintain a rough description of the pose (position and orientation) of each robot, at least relative to the other robots, and we must update those pose estimates as the team moves. This is especially challenging for team members which are not part of the leading edge (and therefore do not necessarily provide image input to the human operator). This will require some aspects of SLAM. Furthermore, in order to determine the leading edge, relative SLAM is required.

The SLAM literature distinguishes between "metric maps" which describe the environment by distances between points [8, 9, 10], and those that use "topological" information[11, 12, 13]. The former approach becomes quickly overwhelmed by computational complexity, but the latter strategies are not particularly accurate. Our approach will be to represent the pose of the individual robots by particle filter-like representation [14, 10] for the pose, which will allow a probabilistic representation for the pose. Interactions between robots will be described by a topologic map, allowing the incorporation of metric information[15, 16, 13].

5. Results

5.1. **Visibility.** In this section, we present derivations which show how to view the entire environment around the robot team. We show that it is possible to have complete coverage outside the team provided at least some subset of robots forms a convex polygon. These theoretical results are presented here. Experiments have confirmed the effectiveness of these methods, using Blender simulation.

Assume we are to operate a team of holonomic robots. For now, assume the operational terrain is approximately planar and horizontal. Each robot is equipped with a single camera which can pan and/or zoom. We model these cameras with pinhole cameras with adjustable focal length. We are concerned with camera formations which maintain "complete external visibility." Define ∂ to be the convex hull of the set of cameras.

Definition An *ray* is a vector in the ground plane, passing through a camera center and through the finite focal plane of that camera, and an *exterior ray* is a ray into the exterior of ∂.

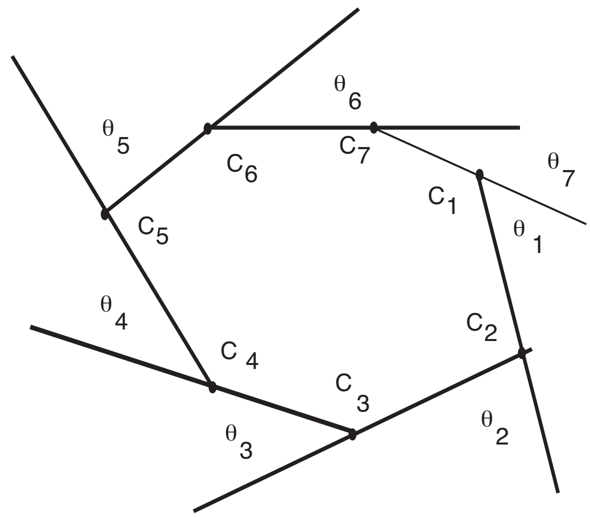

FIGURE 1. A set of 7 cameras defining a convex polygon. Camera i is denoted $\mathbf{c}_i$, and has angular field of view of θ_i.

Definition A set of cameras has *complete external visibility* if there exists an exterior ray in every possible direction. It should be noted that our use of the word "visibility" is different from that of Flocchini et al. [17] who were also interested in a ring of sensors. In their terminiology, "complete visibility" means every sensor can see every other sensor, whereas our definition means the collection of sensors can see everything (that is not occluded) outside the ring.

We will examine a particular arrangement of cameras in this context. Only cameras which are on the convex hull will be of interest. That set (also denoted by ∂), defines a convex polygon such as the one shown in Figure 1. In this example, $\partial = \{c_1, c_2, \cdots, c_7\}$ and $|\partial| = 7$. At each camera c_i, the exterior angle is shown and denoted θ_1. We will also use the terminology c_1 later in this paper to denote the coordinates of camera i.

Theorem 1 The subset of cameras making up the convex hull of a set of cameras provides complete exterior visiblity if the field of view of each camera is the external angle of the polygon at that point.

Proof. Consider the two cameras, c_5 and c_6, shown in Figure 1. Since θ_5 is the angular field of view of camera 5, and θ_6 is similarly the FOV of camera 6, and the FOV's do not overlap, then camera 5 and 6 together observe all angles between 0 and $\theta_5 + \theta_6$. By induction, and since the sum of the exterior angles of a polygon is 2π, then every direction is observed. $\square$

Using a similar argument, we note that each direction is observed only once.

Algorithm 1, Generating complete external coverage

(1) Compute the convex hull of the set of cameras. This problem is $O(n\ log\ n)$ where $n = |\partial|$.
(2) For camera i, orient the camera so the left side of its FOV is colinear with the vector to camera $i - 1$. If $i = 1$, align with camera n.

(3) set the focal length of camera i so that the right side of its FOV is colinear with the vector to camera $i+1$. if $i=n$, align with camera 1.

Algorithm 1 will provide complete coverage, but there is a problem: changing the FOV zooms the observed image, so objects in the scene will appear to grow or shrink in comparison to similar objects at a similar range viewed by other cameras.

5.2. **Correction for different camera focal lengths.** The fact that each camera potentially has a different focal length leads to the following situation: Camera i views half of an object, and camera $i+1$ views the other half. In a composite image, the two halves would potentially different sizes. This is guaranteed to happen unless all the cameras have the same focal length; which can only happen if the contour is a regular polygon. So, to make it appear that the same object has the same size, even though the focal length has changed, we must resample to focal plane, so the the angle subtended by a single pixel is a constant. We accomplish that resampling by considering the zoom-in and zoom-out as separate cases.

First let there be a "standard" focal length for each camera. This might, for example, be the focal length that produces a FOV of $\pi/4$. Then all cameras will have focal lengths that are relative to this.

5.2.1. *Zoom-out.* Figure 2 illustrates the zoom-out case; shortening the focal length. The width of the focal plane in standard position is known. For example, it might be 17.5mm. Similarly, the focal length f_0 and FOV, θ_o, are known. After the zoom, only the FOV, θ is known. From the small right triangle, we have

$$x = x_0 - (f_0 - f)\tan\theta_0 \tag{1}$$

So the size ratio of a particular object on the focal plane is x/x_0, and

$$\frac{x}{x_0} = 1 - \frac{(f_0 - f)}{x_0}\tan\theta_0 \tag{2}$$

But in general, we don't have f available, and must use θ, so the pixel contraction is

$$\alpha = \frac{x}{x_0} = 1 - \frac{(f_0 - \frac{x_0}{\tan\theta})}{x_0}\tan\theta_0 \tag{3}$$

5.2.2. *Zoom In.* In this case, $x_0 = f\tan\theta$, and therefore $f = \frac{x_0}{\tan\theta}$, but

$$dx = f\tan\theta_0 - x_0 \tag{4}$$

$$= \frac{x_0}{\tan\theta}\tan\theta_0 - x_0 \tag{5}$$

(6)

and the pixel dialation is

$$\alpha = \frac{x_0 + dx}{x_0} = \frac{\tan\theta_0}{\tan\theta} \tag{7}$$

Observation: For $\theta \to 0$, using equation 7, the dilation goes to infinity, and the entire scene is viewed in a single pixel. For $\theta \to \inf$, using equation 3, the contraction goes to zero, and every pixel views the entire scene.

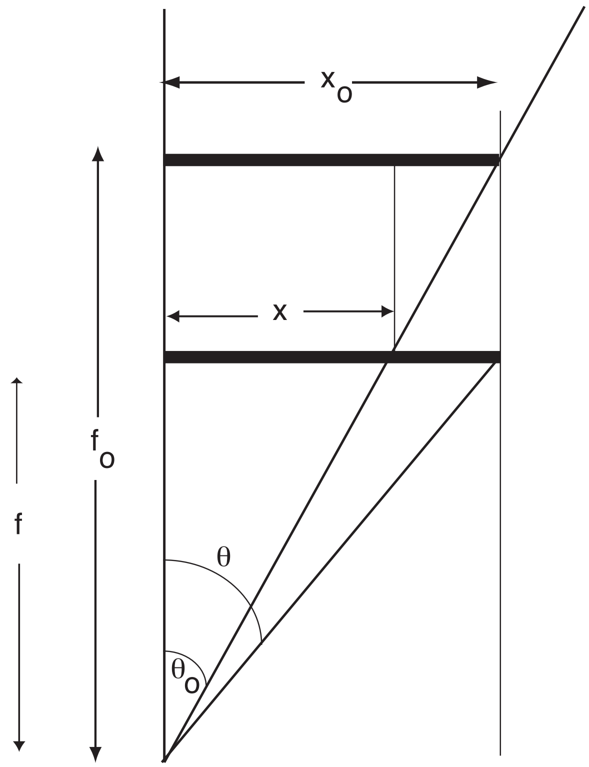

FIGURE 2. Zoom-out: the focal length is shortened from f_0 to f, moving the focal plane closer to the focal point. Before the zoom, the length of the focal plane was $2x_o$, subtending an angle of $2\theta_0$. (only half of the focal plane is shown in this right triangle. After the zoom, the same portion of the visual field only occupies a length of x on the focal plane. So the camera can see more, but a given object occupies fewer pixels on the focal plane.

Algorithm 2, resampling to make objects appear the same size
For i varying from 1 to the number of cameras:

(1) Determine the direction vector to the next camera, $v = \frac{c_{i+1} - c_i}{||c_{i+1} - c_i||}$.
(2) Determine the direction vector from the previous camera, $v' = \frac{c_i - c_{i-1}}{||c_i - c_{i-1}||}$.
(3) Compute θ_i from the inner product, $\theta_i = \cos^{-1} \langle v, v' \rangle$.
(4) Resample the focal plane depending on whether θ_i is a zoom in or a zoom out:

- If $\theta < \theta_0$ (zoom out); in this case, $\alpha < 1$.
  (a) Let the original image have R rows and C columns, and be denoted $f(r, c), \quad r = 1, \cdots, R, c = 1, \cdots, C$.
  (b) Create a new image, $f'(r, c), \quad r = 1, \cdots, \alpha R, c = 1, \cdots, \alpha C$
  (c) Scan over the new image, computing pixel values using for $r = 1$ to $R/\alpha, c = 1, C/\alpha$,

(8) $$f'(r, c) = \Gamma(f, \alpha r, \alpha c) ,$$

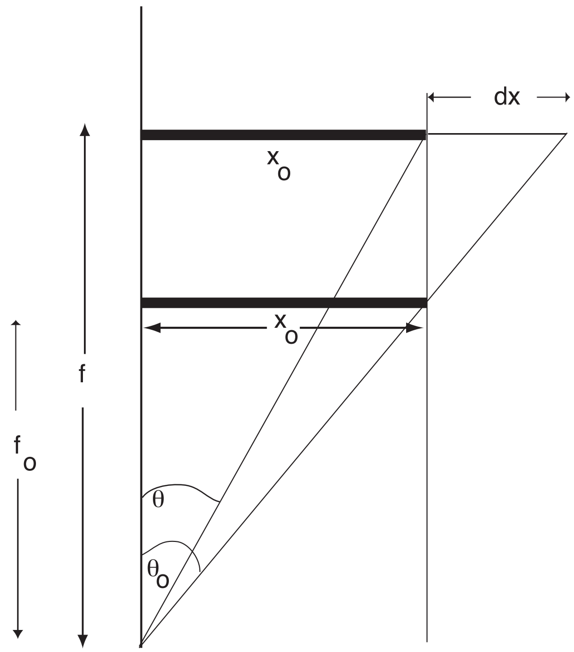

FIGURE 3. Zoom-in: the focal length is lengthened from f_0 to f, moving the focal plane farther from the focal point. The length of the physical focal plane is $2x_o$, subtending an angle of $(2\theta_0)$. (Typically, $2\theta_0$ might be 35mm.) Before the zoom, if we were to image a point at distance f, we would see an additional width of dx. However, when we zoom, and actually move the focal plane to f, we only image a range of x_0, a reduction in the field of view.

where $\Gamma(f, y, x)$ is an interpolation function which finds the best estimate of the discretely sampled image f at the point y, x.

- If $\theta > \theta_0$ (zoom in); in this case, $\alpha > 1$.
  (a) Let the original image have R rows and C columns, and be denoted $f(r, c), \quad r = 1, \cdots, R, c = 1, \cdots, C$.
  (b) Create a new image, $f'(r, c), \quad r = 1, \cdots, \alpha R, c = 1, \cdots, \alpha C$
  (c) Scan over the new image, computing pixel values using for $r = 1$ to $R/\alpha, c = 1, C/\alpha$,

(9) $$f'(r, c) = \Gamma(f, r/\alpha r, c/\alpha) \ ,$$

where $\Gamma(f, y, x)$ is an interpolation function which finds the best estimate of the discretely sampled image f at the point y, x.

When this process is complete, the new image f' will contain objects whose scale is consistent with the corresponding objects[1] in adjacent images.

[1]Since there is no overlap in viewpoints, the same 3D points will not be imaged by adjacent cameras. However, it is quite possible to have the left side of a house in one image and the right side of the same house in a neighboring image, and the scales must be consistent.

5.3. **Computing the Convex Hull.** The convex hull may be computed in two different ways: one way is $O(n^2)$ in the number of cameras, and one is $O(n \log n)$ – faster but much more complex. code. Since for this application, we had only around ten cameras, we implemented the simpler algorithm.

5.4. **Singular Formations.** From Figure 1 we first observe that the interior of the convex hull is not visible. This is an artifact of the architecture of the formation control and the price we pay for the ability to look in every external direction without duplicate observation vectors. This requires that the hull be strictly convex, for a singular condition occurs if cameras i and $i+1$ are colinear, for any i. Thus, our algorithm for formation control will need to ensure strict convexity, as will be discussed in the next section.

5.5. **Formation Control.** The robots may be modeled as having mass and friction and obeying Newtonian mechanics [18]. The robots move in response to applied forces. Some of these forces are virtual (computed) in order to maintain the formation shape, and some are external to ensure avoidance of obstacles.

5.5.1. *Formation Control Forces.* Let $\mathbf{x}_i, \quad i = 1, \cdots, n$ denote the spatial coordinate vector of camera i. This is a 3-vector, but may be considered a 2-vector if all motion is restricted to the ground plane.

By $\mathbf{x}_0$, we denote the coordinates of the special point of attraction for the swarm. In the absence of other forces, $\mathbf{x}_0$ is the center of a regular polygon of cameras. Each camera, camera i, is attracted to the attractor with an inward force equal to

$$\mathbf{f}_{0,i} = \beta_0(\mathbf{x}_0 - \mathbf{x}_i) \ . \tag{10}$$

We observe that, unlike gravity, this force grows stonger with distance instead of weaker.

Each camera experiences a repulsive force from the other cameras, producing a net repulsive force of

$$\mathbf{f}_{ji} = \sum_{j=1, j\neq i}^{n} \beta_c \frac{\mathbf{x}_i - \mathbf{x}_j}{||\mathbf{x}_i - \mathbf{x}_j||^3} \tag{11}$$

Finally, one other force comes into play to ensure the convexity of the formation, a force to ensure that the formation remains convex. To see this, consider the three cameras illustrated in Figure 4. We wish to apply a force to node i, having coordinates $\mathbf{x}_i$, to prevent it from moving into a position where it is colinear with $\mathbf{x}_{i-1}$ and $\mathbf{x}_{i+1}$. We accomplish this by first finding the point p which is on the line between $\mathbf{x}_{i-1}$ and $\mathbf{x}_{i+1}$ and is as close as possible to $\mathbf{x}_i$. The vector from p to $\mathbf{x}_i$ determines the direction of the force acting on camera i.

Since the two vectors are orthogonal, we have first

$$(\mathbf{x}_{i+1} - \mathbf{x}_{i-1}) \cdot (p - \mathbf{x}_i) = 0 \tag{12}$$

and since the point p lies on the line between the two points, it satisfies

$$p = \delta \mathbf{x}_{i-1} + (1-\delta)\mathbf{x}_{i+1} \tag{13}$$

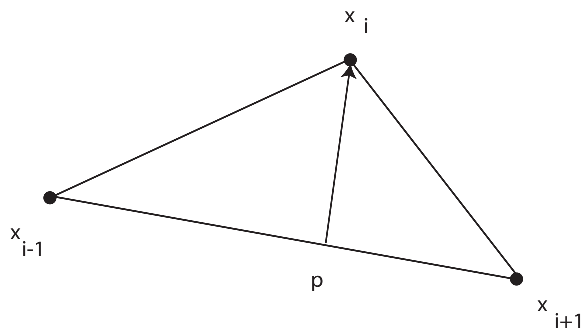

FIGURE 4. Three points on the contour of a formation. To maintain convexity, the center point must not align with the line between its two neighbors.

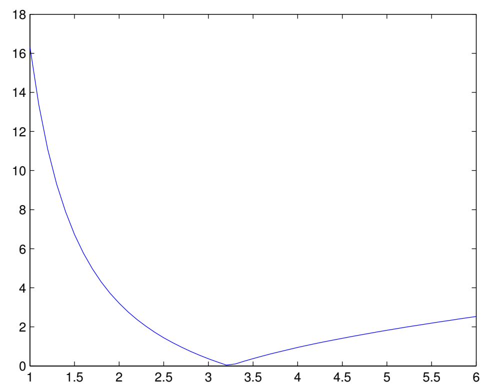

FIGURE 5. The magnitude of the control applied to a typical robot in a ring as a function of the radius of the ring. We observe that the equilibrium state for this rings occurs at r= 3.3.

In the next section, it is shown that δ is determined easily, making it possible to determine p using Equation 13. Then the formation force is defined to be

$$f_{p,i} = \beta pi \frac{\mathbf{p}_i - \mathbf{x}_i}{||\mathbf{p}_i - \mathbf{x}_i||^3} \tag{14}$$

One may think about the three formation forces as artificial forces, or as applied controls. In this paper, we do not attempt to solve the n-body problem in closed form, and instead state that with no other applied forces, we have demonstrated consistetly stable behavior for any initial convex state. Figure 5 shows magnitude of the net control vector on one of the robots in an 8-robot ring, as a function of the radius of the ring.

5.6. **External Forces.** Following the lead of many investigators, we model occlusions by repulsive forces. These forces serve to deform the shape of the ring, to allow passage past individual occlusions, narrow passageways, etc. The restriction is that the ring cannot deform sufficiently to violate strict convexity.

We assume that the spatial coordinates of occlusions and obstacles may be determined by stereopsis[19], or other sensor, and do not address the sensing problem here.

We do make one concession to computational complexity: continuous obstacles are subsampled on the focal plane at a density equal to every k pixels, with a minimum of two points, and only those points are used to generate controls for the ring. Thus, if an object subtends only a few pixels, its control vector is computed very quickly.

We did implement and test one version of a motion model with dynamics, but it proved to be difficult to achieve stability. Analysis of instability proved to come from two sources: 1) this is an n-body problem. Each robot exerts a force on every other robot. There are unexpected minimia, e.g. a symmetric ring of robots with a single robot in the center of the ring. 2) The simulation is heavily computational. In order to achieve simulations in a reasonable time, the step size needs to be large, and the large step size in turn produces unstable behavior.

Since the only purpose of the control simulation is to verify the vision algorithms, we simplified the dynamics to omit inertia and friction, and simply use the model

$$\Delta \mathbf{x}_i = \alpha \mathbf{f}_i \tag{15}$$

At each iteration, each robot simply moved an incremental distance proportional to the applied force. This allowed effective simulation.

5.7. **Demo.** To demonstrate the algorithm, a powerful computer system was set up, utilizing a 6-core I-7, with three NVIDIA graphics processors, controlling a total of five monitors. The operator sits facing the center monitor, with monitors at $\pm 45^o$, and $\pm 90^o$. The user then has the sensation of a full frontal and side view. The display algorithm is demonstrated to be effective and usable.

Videos will be sent under separate cover to the program manager

6. Students Supported

Asif Hussain PhD student, US Permanent Resident
Stuart Heinrich PhD student, US Citizen

7. Publications Resulting from this Grant

To date, the following papers have resulted in whole or in part from this sponsorship.

(1) Amit Bhatia and Wesley Snyder, Pattern Recognition by Cluster Accumuation, IEEE Intelligent Vehicles Symposium (IV 2010), June 2010, San Diego, CA.
(2) Amit Bhatia and Wesley Snyder, Stacked Integral Image, IEEE International Conference on Robotics and Automation, Anchorage, May, 2010.

Appendix A. Derivation of δ

Defining $\mathbf{x}_i = [x_i \; y_i]^T$, and substituting Equation 13 into Equation 12,

$$\begin{bmatrix} x_3 - x_1 \\ y_3 - y_1 \end{bmatrix} = \begin{bmatrix} \alpha x_1 + (1-\alpha)x_3 - x2 \\ \alpha y_1 + (1-\alpha)y_3 - y_2 \end{bmatrix} = 0 \; , \tag{16}$$

leading to

$$\delta = -\frac{y_1y_3 + x_1x_3 + y_2y_3 + x_2x_3 - y_1y_2 - x_1x_2 - x_3^2 - y_3^2}{(x_1 - x_3)^2 + (y_1 - y_3)^2} \tag{17}$$

References

[1] D. Powell, G. Gilbreath, and M. Bruch. Multi-robot operator control unit. In *SPIE Proc. 6230: Unmanned Systems Technology VIII, Defense Security Symposium*, April 2006.

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